Condensed Matter Physics, 200?, Vol. ?, No ?, pp. 1–9

Microscopic construction of the two-fluid model for superfluid helium-4

P. Shygorin, A. Svidzynskyj

Lesia Ukrainka Volyn National University

September 29, 2009

Using a system of Heisenberg's equation of motion for both the normal and the anomalous correlation functions a two-fluid hydrodynamics for superfluid helium-4 was constructed. The method is based on a gradient expansion of the exact equations of motion for correlation functions about a local equilibrium together with explicit use of the local equilibrium statistical operator for superfluid helium in the frame of reference, where condensate is rest.

Key words: *two-fluid hydrodynamics, correlation function, superfluid helium, statistical operator*

PACS: *67.25.dg*

1. Introduction

Superfluid behavior is the most striking property of liquid helium-4. The superfluid ^{4}He is the quantum degenerate system with spontaneous broken symmetry. It's feature is macroscopic occupation the lowest-energy single-particle quantum state or another words is presence of condensate. As a result, the state of statistical equilibrium of the system with spontaneous broken symmetry depends on eight quantities: particle density ρ, energy density ε, momentum density $\vec{j}$ and superfluid velocity $\vec{v}_s$. Presence of additional velocity field leads to that hydrodynamics of the such system is two-fluid.

The two-fluid hydrodynamic equations for the superfluid ^{4}He in the phenomenological consideration were constructed by Landau in 1941 [1]. These equations were derived at the microscopic level by Bogolyubov in 1963 [2].

As a starting point in the Bogolyubov's paper is a set of equation of motion for local quantities (particle density, momentum density and energy density), which easy follows from Heisenberg equations for both creation and annihilation operators; as well as the equation for anomalous average $\langle\psi\rangle$. Thus, from the last follows a hydrodynamic equation for superfluid velocity.

To transition from formal equations of motion to hydrodynamic equations Bogolyubov consider stage of evolution when it is approximate to equilibrium. Then it is possible to assume that in system is established local equilibrium. It described by the statistical operator with parameters that depend on space coordinates. At nearing to the thermodynamic equilibrium these parameters are slowly changed in space and time, therefore their gradients are small. Procedure of expansion by the gradients is formulated by introduction in equations of motion of the so-called "parameter of homogeneity". Then expansion by the gradients coincides with expansion by this parameter. Introduction of the parameter of homogeneity in the Bogolyubov's paper was carried out by a formally way.

When the conservation relations for the local hydrodynamical quantities are constructed, the next step is calculating of a hydrodynamical flows in these equations. The momentum flux Bogolyubov calculate by using very elegant "scale transformation" method. But the flux of energy is obtained inconsistently. More acceptable method calculation of the energy flux using an explicit local equilibrium statistical operator was proposed by Morozov [3].

Our paper imitates the Bogolyubov's article [2], but we work with equations of motion for the correlation functions which are written in the mixed Wigner representation. It allows an expansion by the gradients directly realize, very easy and with rigorous mathematic.

To calculate hydrodynamical flows we are using an explicit form for local equilibrium statistical operator. But in contrast to Morozov's work, which operate with statistical operator of the superfluid helium at the laboratory reference system, we construct one at the reference system in which the condensate is motionless. That gives essential simplification, because in the local frame of reference moving with $\vec{v}_s$ the superfluid component is stopped, then the total current is carried by the normal component.

Construction of the two-fluid model we conditionally separate into two stage. On the first one, using Heisenberg equation of motion for both the normal and the anomalous correlation functions, we derive a conservation relations for densities of particle ρ, momentum $\vec{j}$ and energy ε, as well as equation of motion for superfluid velocity $\vec{v}_s$. On the second stage we express a hydrodynamic flows in conservation relations in terms of already introduced variables (ρ, $\vec{j}$ and ε).

2. Construction of the two-fluid hydrodynamic equations

2.1. Equation of motion for correlation functions in the mixed Wigner representation

The helium-4 is a typical Bose system with pair interaction. It Hamiltonian in the second quantization representation has the next form (we set $\hbar = 1$ throughout this paper)

$$H = \int d\vec{r}\psi^{+}(\vec{r})\left(-\frac{1}{2m}\Delta\right)\psi(\vec{r}) + \frac{1}{2}\int d\vec{r}d\vec{r}'\Phi(\vec{r}-\vec{r}')\psi^{+}(\vec{r})\psi^{+}(\vec{r}')\psi(\vec{r}')\psi(\vec{r}), \quad (1)$$

here $\psi^{+}(\vec{r})$ and $\psi(\vec{r})$ – are the creation and annihilation operators respectively, $\Phi(\vec{r}-\vec{r}') = \Phi(|\vec{r}-\vec{r}'|)$ – interaction potential.

To construct the hydrodynamics of a systems with spontaneous broken symmetry we should proceed from the extended system of correlation functions [5], which is formed both a normal and an anomalous correlation function. Therefore we will start with a system correlation functions in the next form

$$\langle\psi^+(\vec{r}_1,t)\psi(\vec{r}_2,t)\rangle, \quad \langle\psi(\vec{r},t)\rangle. \tag{2}$$

Here the angular brackets indicate an average at the local-equilibrium ensemble, and dependence of the creation and annihilation operators on the time is given through a Heisenberg representation, for instance

$$\psi(\vec{r},t) = e^{\mathrm{i}Ht}\psi(\vec{r})e^{-\mathrm{i}Ht}.$$

Notice, that an average in (2) is treat as quasi-average [4]. For the sake of simplicity, we will not take into account "ν – term" that break a symmetry of the Hamiltonian (1).

Using the Heisenberg's equation of motion

$$\mathrm{i}\frac{\partial\psi(\vec{r},t)}{\partial t} = [\psi(\vec{r},t),H]_- = -\frac{1}{2m}\Delta\psi(\vec{r},t) + \int d\vec{r}'\Phi(\vec{r}-\vec{r}')\psi^+(\vec{r}',t)\psi(\vec{r}',t)\psi(\vec{r},t)$$

we obtain the equations of motion for correlation functions (2).

These equations are as follows

$$\begin{aligned} &\mathrm{i}\frac{\partial}{\partial t}\langle\psi^+(\vec{r}_1,t)\psi(\vec{r}_2,t)\rangle = \frac{1}{2m}\left(\Delta_1-\Delta_2\right)\langle\psi^+(\vec{r}_1,t)\psi(\vec{r}_2,t)\rangle \\ &-\int d\vec{r}'\left\{\Phi(\vec{r}_1-\vec{r}')-\Phi(\vec{r}_2-\vec{r}')\right\}\langle\psi^+(\vec{r}_1,t)\psi^+(\vec{r}',t)\psi(\vec{r}',t)\psi(\vec{r}_2,t)\rangle, \end{aligned} \tag{3}$$

$$\mathrm{i}\frac{\partial}{\partial t}\langle\psi(\vec{r},t)\rangle = -\frac{1}{2m}\Delta\langle\psi(\vec{r},t)\rangle + \int d\vec{r}'\Phi(\vec{r}-\vec{r}')\langle\psi^+(\vec{r}',t)\psi(\vec{r}',t)\psi(\vec{r},t)\rangle. \tag{4}$$

The next step will be a separation of gauge-noninvariant multipliers (in fact we will use a reference system in which the condensate is motionless). Such a separation of phase has the form

$$\psi(\vec{r},t) \to \tilde{\psi}(\vec{r},t) = \psi(\vec{r},t)e^{\mathrm{i}m\chi(\vec{r},t)}.$$

The separation of phase transform a correlation functions by rules

$$\begin{aligned} \langle\psi^+(\vec{r}_1,t)\psi(\vec{r}_2,t)\rangle &= e^{\mathrm{i}m(\chi(\vec{r}_2,t)-\chi(\vec{r}_1,t))}G(\vec{r}_1,\vec{r}_2;t), \\ \langle\psi(\vec{r},t)\rangle &= e^{\mathrm{i}m\chi(\vec{r},t)}F(\vec{r};t), \\ \langle\psi^+(\vec{r}_1,t)\psi^+(\vec{r}',t)\psi(\vec{r}',t)\psi(\vec{r}_2,t)\rangle &= e^{\mathrm{i}m(\chi(\vec{r}_2,t)-\chi(\vec{r}_1,t))}\mathcal{D}^{(1)}(\vec{r}_1,\vec{r}_2,\vec{r}';t), \\ \langle\psi^+(\vec{r}',t)\psi(\vec{r}',t)\psi(\vec{r},t)\rangle &= e^{\mathrm{i}m\chi(\vec{r},t)}\mathcal{D}^{(2)}(\vec{r},\vec{r}';t). \end{aligned}$$

The functions G, F, $\mathcal{D}^{(1)}$ and $\mathcal{D}^{(2)}$ at the statistical equilibrium state is spatial homogeneous. In the nonequilibrium states ones changes, less than more small will be spatial inhomogeneous.

Then the equations of motion for G and F are as follows

$$\begin{aligned}
&\left\{\mathrm{i}\frac{\partial}{\partial t} + m\dot{\chi}(\vec{r}_1,t) - m\dot{\chi}(\vec{r}_2,t)\right\}G(\vec{r}_1,\vec{r}_2;t)\\
&\quad = -\frac{1}{2m}\left[(\hat{\vec{p}}_1 - m\vec{v}_s(\vec{r}_1,t))^2 - (\hat{\vec{p}}_2 + m\vec{v}_s(\vec{r}_2,t))^2\right]G(\vec{r}_1,\vec{r}_2;t)\\
&\quad - \int d\vec{r}'\left\{\Phi(\vec{r}_1-\vec{r}') - \Phi(\vec{r}_2-\vec{r}')\right\}\mathcal{D}^{(1)}(\vec{r}_1,\vec{r}_2,\vec{r}';t), \qquad (5)
\end{aligned}$$

$$\begin{aligned}
&\left\{\mathrm{i}\frac{\partial}{\partial t} - m\dot{\chi}(\vec{r},t)\right\}F(\vec{r};t) = \frac{1}{2m}(\hat{\vec{p}} + m\vec{v}_s(\vec{r},t))^2F(\vec{r};t)\\
&\quad + \int d\vec{r}'\Phi(\vec{r}-\vec{r}')\mathcal{D}^{(2)}(\vec{r},\vec{r}';t). \qquad (6)
\end{aligned}$$

where $\vec{v}_s = \nabla\chi$ is superfluid velocity (velocity of the condensate).

The transition to equations of hydrodynamics is performed using an expansion of equation (5) in terms of space gradients. This expansion can be simply performed by using the so-called mixed Wigner representation [5]. For this purpose, we introduce new variables

$$\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2), \quad \vec{r} = \vec{r}_2 - \vec{r}_1.$$

After the Fourier transformation with respect to relative coordinate $\vec{r}$ we obtain

$$f(\vec{r}_1,\vec{r}_2,t) \to f(\vec{R},\vec{r},t) = \int \frac{d\vec{p}}{(2\pi)^3} f(\vec{R},\vec{p},t)e^{\mathrm{i}\vec{p}\vec{r}},$$

and

$$\begin{aligned}
\vec{r}_1 &\to \vec{R} - \frac{\mathrm{i}}{2}\nabla_{\vec{p}}, & \vec{r}_2 &\to \vec{R} + \frac{\mathrm{i}}{2}\nabla_{\vec{p}},\\
\hat{\vec{p}}_1 &\to \vec{p} - \frac{\mathrm{i}}{2}\nabla_{\vec{R}}, & \hat{\vec{p}}_2 &\to -\vec{p} - \frac{\mathrm{i}}{2}\nabla_{\vec{R}}. \qquad (7)
\end{aligned}$$

Any function of $\vec{R} + \mathrm{i}/2\cdot\nabla_{\vec{p}}$ can be understood in terms of its power-series expansion

$$f(\vec{R} + \frac{\mathrm{i}}{2}\nabla_{\vec{p}}) = f(\vec{R}) + \frac{\mathrm{i}}{2}\frac{\partial f(\vec{R})}{\partial\vec{R}}\frac{\partial}{\partial\vec{p}} - \cdots. \qquad (8)$$

Using procedures (7) and (8) the equations for correlation functions can be written as follows

$$\begin{aligned}
&\frac{\partial G_{\vec{p}}(\vec{R},t)}{\partial t} - m\dot{v}_{si}(\vec{R},t)\frac{\partial G_{\vec{p}}(\vec{R},t)}{\partial p_i} - \frac{\partial}{\partial R_j}\left(\frac{(p_i + mv_{si}(\vec{R},t))^2}{2m}\right)\frac{\partial G_{\vec{p}}(\vec{R},t)}{\partial p_j}\\
&+\left(\frac{p_i}{m} + v_{si}(\vec{R},t)\right)\frac{\partial G_{\vec{p}}(\vec{R},t)}{\partial R_i} + \frac{\partial}{\partial R_j}\left(\frac{1}{2}\int d\vec{r}'\frac{\partial\Phi(r')}{\partial r'_i}r'_j\frac{\partial\mathcal{D}^{(1)}_{\vec{p}}(\vec{R},\vec{r}';t)}{\partial p_i}\right) = 0. \qquad (9)
\end{aligned}$$

$$\left(\mathrm{i}\frac{\partial}{\partial t} - m\dot{\chi}(\vec{R},t)\right)F(\vec{R},t) = \frac{1}{2m}\left(\hat{\vec{p}}^2 + m\vec{v}_s^2(\vec{R},t)\right)F(\vec{R},t)$$
$$-\mathrm{i}\left(\nabla_{\vec{R}} v_{si}(\vec{R},t)\right)F(\vec{R},t) + \int d\vec{r}'\Phi(|\vec{R}-\vec{r}'|)\mathcal{D}^{(2)}(\vec{R},\vec{r}';t). \tag{10}$$

Here

$$G_{\vec{p}}(\vec{R},t) = \int d\vec{r}\langle\psi^+(\vec{R}-\frac{\vec{r}}{2},t)\psi(\vec{R}+\frac{\vec{r}}{2},t)\rangle e^{\mathrm{i}\vec{p}\vec{r}}, \tag{11}$$

$$\mathcal{D}^{(1)}_{\vec{p}}(\vec{R},\vec{r}';t)=\int d\vec{r}\langle\psi^+(\vec{R}-\frac{\vec{r}}{2},t)\psi^+(\vec{r}',t)\psi(\vec{r}',t)\psi(\vec{R}+\frac{\vec{r}}{2},t)\rangle e^{\mathrm{i}\vec{p}\vec{r}}, \tag{12}$$

In the obtained equation (9) the second order terms with respect to space gradient (the terms proportional $\nabla^2_{\vec{R}}$) were neglected.

The equation (9) we denominate as forming equation, because it's using gives a conservation laws for the hydrodynamic quantities. In terms of (9) will be obtained equation of motion for superfluid velocity.

Let's pass to obtaining differential conservation laws (balance equations).

2.2. Equation of motion for superfluid velocity

Let us consider equation for anomalous correlation function (10). After separating real and imaginary parts at this equation we find

$$\left(m\dot{\chi}(\vec{R},t) + \frac{1}{2}m\vec{v}_s^2(\vec{R},t)\right)F(\vec{R},t) = \frac{1}{2m}\nabla^2_{\vec{R}}F(\vec{R},t) - \int d\vec{r}'\Phi(|\vec{R}-\vec{r}'|)\mathcal{D}^{(2)}(\vec{R},\vec{r}';t). \tag{13}$$

Hence

$$\dot{\chi}(\vec{R},t) = -\frac{1}{2}\vec{v}_s^2(\vec{R},t) + \frac{\nabla^2_{\vec{R}}F(\vec{R},t)}{2m^2F(\vec{R},t)} - \frac{\int d\vec{r}'\Phi(|\vec{R}-\vec{r}'|)\mathcal{D}^{(2)}(\vec{R},\vec{r}';t)}{mF(\vec{R},t)},$$

or

$$\dot{\chi}(\vec{R},t) = -\frac{1}{2}\vec{v}_s^2(\vec{R},t) - \frac{\mu(\vec{R},t)}{m}. \tag{14}$$

Here

$$\mu(\vec{R},t) = -\frac{\nabla^2_{\vec{R}}F(\vec{R},t)}{2m^2F(\vec{R},t)} + \frac{\int d\vec{r}'\Phi(|\vec{R}-\vec{r}'|)\mathcal{D}^{(2)}(\vec{R},\vec{r}';t)}{mF(\vec{R},t)}$$

is chemical potential [2].

Applying the operation $\nabla_{\vec{R}}$ to equation (14) we obtain the equation of motion for superfluid velocity

$$m\frac{\partial\vec{v}_s}{\partial t} + \nabla_{\vec{R}}\left(\frac{m\vec{v}_s^2}{2} + \mu\right) = 0. \tag{15}$$

This is the first hydrodynamic equation and shows that the superfluid accelerates freely under the applied fields. The remaining hydrodynamic equations are provided by the conservation relations for the particle density $\rho(\vec{R},t)$, momentum density $\vec{j}(\vec{R},t)$ and energy density $\mathcal{E}(\vec{R},t)$. These equations simply obtain from calculation of moments of forming equation (9).

2.3. Equation for particle density

By definition

$$\rho(\vec{R},t) = m\langle\psi^{+}(\vec{R},t)\psi(\vec{R},t)\rangle = m\int \frac{d\vec{p}}{(2\pi)^3} G_{\vec{p}}(\vec{R},t).$$

After integrating (9) over $\vec{p}$ we find

$$\frac{\partial\rho}{\partial t} + \mathrm{div}\vec{j} = 0, \tag{16}$$

where

$$\vec{j}(\vec{R},t) = \int \frac{d\vec{p}}{(2\pi)^3}\vec{p}G_{\vec{p}}(\vec{R},t) + \rho\vec{v}_s \equiv \vec{j}_0 + \rho\vec{v}_s. \tag{17}$$

The (16) is an equation of continuity for particle density. The $\vec{j}(\vec{R},t)$ is a momentum density respectively, and $\vec{j}_0$ is a momentum density in the reference system where condensate is motionless. Calculation of the $\vec{j}_0$ in explicit form will be performed in the section 3.

2.4. Equation for momentum density

Using definition (17), we find that

$$\frac{\partial j_k}{\partial t} = \frac{\partial}{\partial t}(j_{0k} + \rho v_{sk}) = \int \frac{d\vec{p}}{(2\pi)^3}(p_k + m v_{sk})\frac{\partial G_{\vec{p}}(\vec{R},t)}{\partial t} + \rho\frac{\partial v_{sk}}{\partial t}.$$

Taking the moment of the forming equation (9) with respect to $\vec{p} + m\vec{v}_s$ and using equation of motion for the superfluid velocity (15), we obtain

$$\frac{\partial j_k}{\partial t} + \frac{\partial \Pi_{kj}}{\partial R_j} = 0. \tag{18}$$

The flow of momentum density (stress tensor) is given by

$$\begin{aligned}
\Pi_{kj} &= \frac{1}{m}\int \frac{d\vec{p}}{(2\pi)^3}(p_k + m v_{sk})(p_j + m v_{sj})G_{\vec{p}}(\vec{R},t) \\
&- \frac{1}{2}\int d\vec{r}' \frac{\partial\Phi(r')}{\partial r'_k} r'_j \int \frac{d\vec{p}}{(2\pi)^3}\mathcal{D}^{(1)}_{\vec{p}}(\vec{R},\vec{r}';t) \\
&= v_{sk} j_{0j} + v_{sj} j_{0k} + \rho v_{sk} v_{sj} + \Pi_{0kj},
\end{aligned} \tag{19}$$

where

$$\Pi_{0kj} = \frac{1}{m}\int \frac{d\vec{p}}{(2\pi)^3} p_k p_j G_{\vec{p}}(\vec{R},t) - \frac{1}{2}\int d\vec{r}' \frac{\partial\Phi(r')}{\partial r'_k} r'_j \int \frac{d\vec{p}}{(2\pi)^3}\mathcal{D}^{(1)}_{\vec{p}}(\vec{R},\vec{r}';t). \tag{20}$$

2.5. Equation for energy density

By definition the energy density of particles in the laboratory system of reference is as follows

$$\mathcal{E}(\vec{r},t)=\frac{1}{2m}\langle\nabla\psi^+(\vec{r},t)\nabla\psi(\vec{r},t)\rangle+\frac{1}{2}\int d\vec{r}'\Phi(|\vec{r}-\vec{r}'|)\langle\psi^+(\vec{r},t)\psi^+(\vec{r}',t)\psi(\vec{r}',t)\psi(\vec{r},t)\rangle.$$

In the system of reference where condensate is motionless energy density is

$$\mathcal{E}(\vec{R},t)=\frac{1}{2m}\int\frac{d\vec{p}}{(2\pi)^3}(\vec{p}+m\vec{v}_s)^2G_{\vec{p}}(\vec{R},t)+\frac{1}{2}\int d\vec{r}'\Phi(|\vec{R}-\vec{r}'|)\int\frac{d\vec{p}}{(2\pi)^3}\mathcal{D}^{(1)}_{\vec{p}}(\vec{R},\vec{r}';t),$$

or

$$\mathcal{E}=\mathcal{E}_0+\vec{j}_0\vec{v}_s+\frac{1}{2}\rho v_s^2, \tag{21}$$

where

$$\mathcal{E}_0=\frac{1}{2m}\int\frac{d\vec{p}}{(2\pi)^3}p^2G_{\vec{p}}(\vec{R},t)+\frac{1}{2}\int d\vec{r}'\Phi(|\vec{R}-\vec{r}'|)\int\frac{d\vec{p}}{(2\pi)^3}\mathcal{D}^{(1)}_{\vec{p}}(\vec{R},\vec{r}';t). \tag{22}$$

By analogy to previous subsections we find

$$\frac{\partial\mathcal{E}}{\partial t}+\mathrm{div}\vec{Q}=0. \tag{23}$$

The energy flow is given by

$$\vec{Q}=\left(\mathcal{E}_0+\vec{j}_0\vec{v}_s+\frac{1}{2}\rho v_s^2\right)\vec{v}_s+\frac{1}{2}v_s^2\vec{j}_0+\vec{\Pi}_0\vec{v}_s+\vec{Q}_0, \tag{24}$$

where

$$\begin{aligned}Q_{0k} &= \frac{1}{2m}\int\frac{d\vec{p}}{(2\pi)^3}p^2p_kG_{\vec{p}}(\vec{R},t)+\frac{1}{2m}\int d\vec{r}'\Phi(r')\int\frac{d\vec{p}}{(2\pi)^3}p_k\mathcal{D}^{(1)}_{\vec{p}}(\vec{R},\vec{r}';t)\\ &- \frac{1}{2m}\int d\vec{r}'\frac{\partial\Phi(r')}{\partial r'_j}r'_k\int\frac{d\vec{p}}{(2\pi)^3}p_j\mathcal{D}^{(1)}_{\vec{p}}(\vec{R},\vec{r}';t)\end{aligned} \tag{25}$$

The set of equations (15), (16), (18) and (23) form a complete system of two-fluid hydrodynamic equations for superfluid helium-4.

3. Calculation of the hydrodynamical flows

In the previous section was obtained a system of balance equations. These equations unlocked, because the flows (17), (20) and (25) is unknown. When we have an explicit expression for G-function, then finding of the hydrodynamical flows is realizing by calculation of the momentum integrals. In the case of superfluid helium finding of the G-function is impossible. Therefore we must develop some "indirect" method for finding of the flows (17), (20) and (25).

In this article for finding of the hydrodynamical flows we used an explicit expression for the local equilibrium statistical operator. In contrast to paper by Morozov [3] we construct statistical operator in the system of reference where condensate is motionless, that leads some simplification.

The local equilibrium statistical operator that describe a superfluid helium in the system of reference where condensate is motionless is as follows

$$\hat{\varrho} = \exp\left\{ \int d\vec{r}\beta(\vec{r}) \left[\Omega(\vec{r}) - \hat{H}_0(\vec{r}) - \vec{u}\hat{\vec{P}}_0(\vec{r}) - \frac{\mu}{m}\hat{\rho}(\vec{r}) \right] \right\}. \tag{26}$$

In the local frame of reference moving with $\vec{v}_s$ the superfluid component is stopped, then the total current is carried by the normal component.

Therefore

$$\vec{j}_0 =< \hat{\vec{P}}_0 >= -\frac{\partial \Omega}{\partial \vec{u}} \equiv \rho_n \vec{u}. \tag{27}$$

Here ρ_n is the normal fluid density.

Substituting from (27) into (17) we find the momentum density (mass flow)

$$\vec{j} = \vec{j}_0 + \rho\vec{v}_s = \rho_n(\vec{v}_n - \vec{v}_s) + \rho\vec{v}_s = \rho_n\vec{v}_n + (\rho - \rho_n)\vec{v}_s \equiv \rho_n\vec{v}_n + \rho_s\vec{v}_s, \tag{28}$$

where $\rho_s = \rho - \rho_n$ is superfluid density.

To find the stress tensor we use a very elegant "scale transformation" method introduced by Bogolyubov [2]. Simple calculation gives

$$\Pi_{0ik} = \rho_n u_i u_k + \delta_{ik} P, \tag{29}$$

where $P = \rho \dfrac{\partial \Omega}{\partial \rho}$ – is a pressure.

The final form of a stress tensor is

$$\Pi_{ik} = \Pi_{0ik} + v_{si} j_{0k} + v_{sk} j_{0i} + \rho v_{si} v_{sk} = \rho_s v_{si} v_{sk} + \rho_n v_{ni} v_{nk} + \delta_{ik} P. \tag{30}$$

To calculate the energy flux we employ the obvious identity [3]:

$$\langle [A, \hat{S}] \rangle = 0, \tag{31}$$

where A is some dynamic quantities and $\hat{S}$ is an entropy operator, that defined by relationship $\hat{\varrho} = \exp\{-\hat{S}\}$.

Substituting in (31) $A = H$ and using (26), (29) we find

$$\vec{Q}_0 = \left(\rho_n u^2 + \frac{\rho\mu}{m} + TS \right) \vec{u}. \tag{32}$$

Finally, expressions for hydrodynamical flows has the form:

$$\begin{aligned} \vec{j} &= \rho_s\vec{v}_s + \rho_n\vec{v}_n, \quad \vec{v}_n \equiv \vec{u} + \vec{v}_s, \\ \Pi_{ik} &= \rho_n v_{ni} v_{nk} + \rho_s v_{si} v_{sk} + \delta_{ik} P, \\ \vec{Q} &= \left(\frac{v_s^2}{2} + \frac{\mu}{m} \right) \vec{j} + TS\vec{v}_n + \rho_n \vec{v}_n (\vec{v}_n \cdot (\vec{v}_n - \vec{v}_s)). \end{aligned} \tag{33}$$

These hydrodynamical flows are coincide with ones in two-fluid hydrodynamics of Landau [1].

Acknowledgements

The work was supported by the State Foundation for Fundamental Research of Ukraine (Project No. F25.2/011).

References

1. Landau L. D. JETF, 1941, **11**, 592 (in Russian).
2. Bogolyubov N. N. To the question about hydrodynamics of superfluid liquid. Preprint UINR, Dubna, 1963 (in Russian).
3. Morozov V. G. Teor. Mat. Fiz., 1976, **28**, 267 (In Russian).
4. Bogolyubov N. N., Bogolyubov N. N. (ju.) Introduction to the quantum statistical mechanics. Moscow, Nauka, 1984 (in Russian).
5. Svidzynsky A.V. Microscopic theory of superconductivity. Vol. 1. Lutsk, Vezha, 1999 (in Ukrainian).

Мікроскопічна побудова дворідинної моделі для надплинного гелію

П. Шигорін, А. Свідзинський

Волинський національний університет ім. Лесі Українки

29 вересня 2009 р.

Використовуючи систему гайзенбергівських рівнянь руху для нормальної та аномальної кореляційних функцій було побудовано дворідинну гідродинаміку для надплинного гелію-4. Виведення засноване на розкладі за градієнтами точних рівнянь руху для кореляційних функцій поблизу локальної рівноваги разом з використанням явного вигляду для локально-рівноважного статистичного оператора для надплинного гелію в системі відліку, де конденсат нерухомий.

Ключові слова: Дворідинна гідродинаміка, кореляційна функція, надплинний гелій-4, статистичний оператор

PACS: 67.25.dg