On automorphisms of the semigroup of order-decreasing order-preserving full transformation of the boolean

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Let $(M, \leq)$ be a poset. A transformation $\alpha : M \to M$ is called *order-decreasing*, if $\alpha(x) \leq x$ for all $x \in M$. The set $\mathcal{F}(\mathcal{M})$ of such transformations is a semigroup with respect to the composition of transformations. A transformation α is called *order-preserving*, if for every $x, y \in M$, $x \leq y$ implies $\alpha(x) \leq \alpha(y)$. The set of such transformations forms a semigroup, which is denoted by $\mathcal{O}(\mathcal{M})$. The intersection $\mathcal{C}(\mathcal{M}) = \mathcal{F}(\mathcal{M}) \cap \mathcal{O}(\mathcal{M})$ is called the semigroup of *order-decreasing order-preserving* transformations of the M.

Many authors studied semigroups $\mathcal{F}(\mathcal{M})$, $\mathcal{O}(\mathcal{M})$ and $\mathcal{C}(\mathcal{M})$ in the case where the poset M is a finite chain. The analogues of these semigroups were studied also for all partial transformations or all partial injective transformations of the set M. These semigroups for other posets are studied much worse.

We consider a semigroup $\mathcal{C}(\mathcal{B}_n)$ were $\mathcal{B}_n$ is the set of all subsets of a n-element set naturally ordered by inclusion. Note that the semigroup $\mathcal{F}(\mathcal{B}_n)$ of order-decreasing transformations of the boolean $\mathcal{B}_n$ was studied in [3].

A number $h_\alpha = \sum_{A \in im(\alpha)} [\varnothing, A]$ is called *the height* of an element $\alpha \in \mathcal{C}(\mathcal{B}_n)$.

Theorem 1. *Every automorphism of the semigroup $\mathcal{C}(\mathcal{B}_n)$ preserves the height of elements.*

Theorem 2. *The automorphism group of the semigroup $\mathcal{C}(\mathcal{B}_n)$ is isomorphic to the symmetric group S_n.*

Note that the automorphism group of the semigroup $\mathcal{IO}_n$ of order-preserving injective transformations of n-element chain was described in [2].

References

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