

Existence of periodical solutions of the Duffing equation with impulsive effects

Valentyn Sobchuk*, Ph. D. student, Kyiv National University.

Kuanysh Yelgondyev, Doctor, Kyiv National University.

ABSTRACT

We study nonlinear differential Duffing equation

$$\ddot{x} + \omega^2 x = \delta x^3, \quad (1)$$

with impulsive effects [1]

$$\Delta \left. \frac{dx}{dt} \right|_{x=x_*} = \left. \frac{dx}{dt} \right|_{t=t_*+0} - \left. \frac{dx}{dt} \right|_{t=t_*-0} = I(\dot{x}). \quad (2)$$

Impulsive effects in the system (1), (2) occur at the moments of time when the moving point passes some fixed position $x = x_*$. The impulsive influences instantly increase the velocity of the point by the value $I(\dot{x})$.

The solution of the problem (1), (2) is a function $x(t)$ continuous on $t \in \mathbb{R}$ and continuously differentiable on t except the moments of impulsive effects, where its derivative $\dot{x}(t)$ is continuous from the right side.

Qualitative analysis of behaviour of phase trajectories for the problem (1), (2) is fulfilled. Necessary and sufficient conditions for existence of periodic solutions are obtained, and stability of the periodic solutions is studied.

The examples of function $I(\dot{x})$ in (2) of impulsive effects are given for which the problem (1), (2) has the periodic solutions.

References

1. Samoilenko A.M., Perestyuk N.A. Impulsive differential equations. World Scientific series on nonlinear science, Series A, Vol. 14, 1995.
2. Samoilenko A.M., Samoilenko V.Hr., Sobchuk V.V. On periodic solution of equation of mathematical pendulum with impulsive effects. Ukrainian Math. J. 1999, v. 51, n. 6, pp. 827–834.

Keywords: *Duffing equation, impulsive effects*

Mathematics Subject Classification: *93E20, 93C60*

Contact Address: vsam@imath.kiev.ua